Documentation of the MATLAB function `covfunfactor.m`

The function's arguments and returned quantities are listed in the help section of the function which is reproduced below.

```
function [phi,eps]=covfunfactor(C,m,p,maxiter,tol)
% given the coefficients of a finite mxm covariance function of order p solves for the
% coefficients of the mxm polynomial factor phi from the equn. phi(z)*phi(z^-1)' = C(z)
% using an iterative scheme.
% The maximum no. of iterations (maxiter) or the required accuracy of the solution (tol)
% can optionally be specified else default values are used.
% The solution accuracy is described by the maximum equation discrepancy which is returned as eps
% For uniqueness of solution the coefficient phi_0 is constrained to be upper triangular
```

To be specific, the given quantity `C` has elements `C(i,j,k+1)` which are coefficients $C_{i\,j\,k}$ for $i, j = 1 \ldots m$ and $k = 0 \ldots p$ of matrices C_k. The returned quantity `phi` similarly has elements `phi(i,j,k+1)` which are coefficients $\phi_{i\,j\,k}$ for $i, j = 1 \ldots m$ and $k = 0 \ldots p$ of matrices ϕ_k.

The function attempts to solve the equations:

$$C_k = \sum_{v=0}^{p-k} \phi_{k+v}\phi_v', \quad k = 0 \ldots p$$

which are equivalent to solving the functional equation

$$C(z) = \phi(z)\phi(z)'$$

where

$$C(z) = \sum_{k=-p}^{p} C_k z^k$$

taking $C_k = C_{-k}$ for negative k, and

$$\phi(z) = \sum_{k=0}^{p} \phi_k z^k.$$

To ensure a unique solution, the coefficient ϕ_0, which is the matrix with elements `phi(i,k,1)`, is constrained to be upper triangular. Setting $\theta(z) = \phi_0^{-1}\phi(z)$ the functional equation can also be uniquely expressed

$$C(z) = \theta(z)V\theta(z)'$$

where $V = \phi_0\phi_0'$ and $\theta_0 = I$.

The equations only have a solution if $C(z)$ is non-negative everywhere on the unit circle. This condition is not checked in the function, but its failure will be evidenced by a large value of the returned scalar `eps` indicating lack of convergence of the iterations. The default value of `maxiter` is 10 and that of `tol` is 10^{-10}.

Further details of the equations used in the iterations are given in Tunnicliffe Wilson (1972) and the convergence properties of the method are demonstrated in Tunnicliffe Wilson (1978).

References

G. Tunnicliffe Wilson. The factorization of matricial spectral densities. *SIAM Journal on Applied Mathematics,*, 23:420–426, 1972.

G. Tunnicliffe Wilson. A convergence theorem for spectral factorization. *Journal of Multivariate Analysis*, 8:222–232, 1978.