Documentation of the MATLAB function `SSfilsmoMV.m`

The function's arguments and returned quantities are listed in the help section of the function which is reproduced below.

```
function [stestfil,stestsmo,stvarfil,stvarsmo] =SSfilsmoMV(x,H,T,G,P)
% State Space model filtering and smoothing of non-time-varying model
% using square root methods for stable forward and backward computations
% estimates filtered states and smoothed states with their variance matrices
% for m by n series x, which can incorporate missing values set as NaN,
% represented by a state space model with observation and transition matrices H,T
% and disturbance variance matrix G'G supplied via G
% with no observation noise (which could be incorporated into states).
% Optionally the stationary state variance matrix P'P can be supplied via P
% else it is approximated in the function
% stestfil is estimate given all observations up to and including that time
% stestsmo is estimate given all observations
% stvarfil is filtered state estimate variance matrix
% stvarsmo is smoothed state estimate variance matrix
```

This function forms the filtered and smoothed estimates of the states of a specified state space model given an observed series represented by the model.

The given array x, whose size $m \times n$ is determined in the function, has elements `x(i,t)` which are the values $x_{i,t}$ for $i = 1 \ldots m$, $t = 1 \ldots n$, of a times series x_t. Some of the elements of this array may be set to `NaN` as indicating an unknown or missing observation.

This series is represented in terms of state vectors S_t of dimension d by the model observation equation

$$x_t = H\, S_t. \tag{1}$$

where H is a fixed (i.e. non-time-varying) observation matrix of size $m \times d$ which is supplied in the array `H`. The state vector evolves according to the model transition equation

$$S_t = T\, S_{t-1} + E_t, \tag{2}$$

where T is a fixed square transition matrix of size d. The dimension d is determined in the function from the size of T which is supplied in the array `T`.

The supplied array `G` holds the elements of the $b \times d$ matrix G which determines the variance of the model disturbance term E_t according to

$$\mathrm{Var}\, E_t = G'\, G.$$

The dimension b need not be equal to d.

The optionally supplied array `P` holds the elements of the $a \times d$ matrix P which determines the variance of the initializing state vector X_0 according to

$$\mathrm{Var}\, X_0 = P'\, P.$$

The dimension a need not be equal to d. If `P` is not supplied, its value is generated by (implicitly) running the transition equation (2) for approximately 1000 time steps starting from a state of value and variance zero. If (2) represents a stationary process, i.e. all the eigenvalues of T have

magnitude less than unity, P will then correspond to the stationary process variance. If the process is not stationary, P may become very large, corresponding to ignorance of the initial state.

The use of G and P to specify the variances of E_t and X_t is consistent with the use of "square roots" of variances throughout the computations, in order to improve numerical precision and stability.

The elements `stestfil(i,t)` for $i = 1 \ldots m$ and $t = 1 \ldots n$ of the returned array `stestfil` hold the filtered estimates of the states $S_{i,t}$. These are, for each value of t, the minimum error variance linear estimates given the known (non-missing) observations of $x_1, x_2, \ldots, x_t$, *up to* time t.

The elements `stvarfil(i,j,t)` for $i, j = 1 \ldots m$ of the returned array `stvarfil` hold, for each $t = 1 \ldots n$, the elements of the error variance matrices of the filtered estimates of the state vector S_t.

The elements `stestsmo(i,t)` for $i = 1 \ldots m$ and $t = 1 \ldots n$ of the returned array `stestsmo` hold the smoothed estimates of the states $S_{i,t}$. These are, for each value of t, the minimum error variance linear estimates given *all* the known (non-missing) observations of $x_1, x_2, \ldots, x_n$, up to time n.

The elements `stvarsmo(i,j,t)` for $i, j = 1 \ldots m$ of the returned array `stvarsmo` hold, for each $t = 1 \ldots n$, the elements of the error variance matrices of the smoothed estimates of the state vector S_t.

Any missing values in the array x are *not* replaced by estimates.

A document describing the methods used in this function is given in the Algorithms/Technical pages for Chapter 2.