Documentation of the MATLAB function `PredCoef.m`

The function's arguments and returned quantities are listed in the help section of the function which is reproduced below.

```
function [phi,ivs,psi,pvs,phir,ivsr,phikk,phikkr]=PredCoef(r0,r,s0,s,m,h,K)
%
% given the covariance function of an m-dim times series forms forward and backward one
% step prediction coefficients and prediction error variances from its own past
using the Whittle recursions
% also, if h>0, forms prediction coefficients and error variances for a
further h-dim series from the
% past of the given series from its cross-covariances with that series
% set h=0 if no further series relevant: then s0 and s can be set to zero
and psi and pvs are returned as zero
% r0, r are the mxm variance and auto-covariances from lag 1 to at least K
of the given series (m-dim) x
% s0, s are the hxh variance and hxm cross-covariances from lag 1 to at least K
of the further series (h-dim) y with x
% phi are -ve (m,m) lag K prediction coefficients of x on its lagged (1 to K) values,
prefixed by I
% phikk are lag k prediction coefficients for order k = 1 to K predictors,
prefixed by 0
% phir are -ve (m,m) lag K backward prediction coefficients of x
on its lagged (1 to K) values, prefixed by I
% phikkr are lag k prediction coefficients for order k = 1 to K backward predictors
prefixed by 0
% ivs are innovation variances of x for AR orders (1 to K), prefixed by r0.
% ivsr are backward innovation variances of x for AR orders (1 to K), prefixed by r0.
% psi are (hxm) prediction coefficients of variable y on lagged (1 to K) values of x
prefixed by zero
% pvs are (hxh) prediction error variances of y on lagged values of x (1 to K)
prefixed by s0
```

To be specific, the given array `r0` has elements `r0(i,j)` which are the covariances $\mathrm{Cov}(x_{i,t}, x_{j,t})$ for $i, j = 1 \ldots m$,, and the array `r` has elements `r(i,j,k)` which are the covariances $\Gamma_{x\,x\,i,j,k}$ at lag k, defined for a stationary time series x_t by

$$\Gamma_{x\,x\,i,j,k} = \mathrm{Cov}(x_{i,t}, x_{j,t-k}),$$

for $i, j = 1 \ldots m$ and $k = 1 \ldots L$ where $L >= K$.

Further, the given array `s0` has elements `s0(i,j)` which are the covariances $\mathrm{Cov}(y_{i,t}, y_{j,t})$ for $i, j = 1 \ldots h$, and the array `s` has elements `s(i,j,k)` which are the covariances $\Gamma_{y\,x\,i,j,k}$ at lag k, defined between stationary times series y_t and x_t by

$$\Gamma_{y\,x\,i,j,k} = \mathrm{Cov}(y_{i,t}, x_{j,t-k}),$$

for $i = 1 \ldots h$, $j = 1 \ldots m$ and $k = 1 \ldots L$ where $L >= K$.

The returned array `phi` with elements `phi(i,j,k+1)` for $i, j = 1 \ldots m$ and $k = 1 \ldots K$ contains the coefficients $\phi_{i,j,k}$ of the matrices ϕ_k which minimize the variance of the prediction error $e_{t,K}$ of x_t in terms of $x_{t-1} \ldots x_{t-K}$:

$$e_{t,K} = x_t + \phi_1 x_{t-1} + \cdots + \phi_K x_{t-K}. \tag{1}$$

The elements of `phi(i,j,1)` are set to those of the identity matrix.

The returned array `ivs` with elements `ivs(i,j,k+1)` for $i, j = 1 \dots m$ contains the coefficients $V_{i,j,k}$ of the matrices $V_k = \operatorname{Var} e_{t,k}$ for each value of $k = 0 \dots K$, taking $V_0 = \operatorname{Var} x_t$.

The returned array `phikk` with elements `phikk(i,j,k+1)` for $i, j = 1 \dots m$ contains, for each $k = 1 \dots K$, the coefficients $\phi_{i,j,k}$ of the matrix ϕ_k in the expression (1) when the order K of the predictor is set equal to k. The elements of `phikk(i,j,1)` are set to zero.

This array, together with the array of backward coefficients in `phikkr`, is useful for computing the quantities referred to as the partial R^2 in Section 2.9 of the book, with an illustration plotted in Fig. 2.9.

The returned array `phir` with elements `phir(i,j,k+1)` for $i, j = 1 \dots m$ and $k = 1 \dots K$ contains the coefficients $\tilde{\phi}_{i,j,k}$ of the matrices $\tilde{\phi}_k$ which minimize the variance of the backward or reversed time prediction error $\tilde{e}_{t-K-1,K}$ of x_{t-K-1} in terms of $x_{t-K} \dots x_{t-1}$:

$$\tilde{e}_{t-K-1,K} = x_{t-K-1} + \tilde{\phi}_1 x_{t-K} + \cdots + \tilde{\phi}_K x_{t-1}. \tag{2}$$

The elements of `phir(i,j,1)` are set to those of the identity matrix.

The returned array `ivsr` with elements `ivsr(i,j,k+1)` for $i, j = 1 \dots m$ contains the coefficients $\tilde{V}_{i,j,k}$ of the matrices $\tilde{V}_k = \operatorname{Var} \tilde{e}_{t-k-1,k}$ for each value of $k = 0 \dots K$, taking $\tilde{V}_0 = \operatorname{Var} x_t$.

The returned array `phikkr` with elements `phikkr(i,j,k+1)` for $i, j = 1 \dots m$ contains, for each $k = 1 \dots K$, the coefficients $\tilde{\phi}_{i,j,k}$ of the matrix $\tilde{\phi}_k$ in the expression (2) when the order K of the predictor is set equal to k. The elements of `phikkr(i,j,1)` are set to zero.

The returned array `psi` with elements `psi(i,j,k+1)` for $i = 1 \dots h$, $j = 1 \dots m$ and $k = 1 \dots K$ contains the coefficients $\psi_{i,j,k}$ of the matrices ψ_k which minimize the variance of the prediction error $f_{t,K}$ of y_t in terms of $x_{t-1} \dots x_{t-K}$:

$$f_{t,K} = y_t + \psi_1 x_{t-1} + \cdots + \phi_K x_{t-K}. \tag{3}$$

The elements of `psi(i,j,1)` are set to zero.

The returned array `pvs` with elements `pvs(i,j,k+1)` for $i, j = 1 \dots h$ contains the coefficients $W_{i,j,k}$ of the matrices $W_k = \operatorname{Var} f_{t,k}$ for each value of $k = 0 \dots K$, taking $W_0 = \operatorname{Var} y_t$.